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Assignment 2
Comp 510
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19.1-1
Suppose that x is a node in a binomial tree within a binomial heap, and assume that sibling[x] != NIL. If x is not a root, how does degree[sibling[x]] compare to degree[x]? How about if x is a root?

The degree of the sibilings is always one more than the degree of x, as determined by lemma 19.1, so we have degree[sibiling[x]] = degree[x] – 1.

Due to the definition of binomial heaps, if x is a root, its sibilings are in order of increasing degree, giving us the fact that degree[sibiling[x]] > degree[x].

19.1-2
If x is a nonroot node in a binomial tree within a binomial heap, how does degree[x] compare to degree[p[x]]?

We can determine that x is the child (or child of child of..etc.) of p[x]. Therefore if x is the first child of p[x], then degree[p[x]] = degree[x] + 1.

19.1-3
Suppose we label the nodes of binomial tree Bk in binary by a postorder walk, as in Figure 19.4. Consider a node x labeled l at depth i, and let j=k-i. Show that x has j 1's in its binary representation.

Given that this is correct for l being at depth i = 0, (B_0 = 1111), we know that l at depth i+1 will have one more 1 bit. Thus this relationship holds for the entire tree.

How many binary k-strings are there that contain exactly j 1's?

Looking at figure 19.4, we can see that there are k choose i bitstrings with j number of 1 bits.

Show that the degree of x is equal to the number of 1's to the right of the rightmost 0 in the binary representation of l.

From the definition of binomial heaps, and its listing of sibilings in increasing order, we can see this pattern in 19.4. That is, in the picture, 1110 has degree 0, 1101 has degree 1 (one 1 to the right of the right most 0. This corresponds to degree[x], as explained previously in 19.1-1.

References

Second edition of the textbook (from what I could read on google)
http://integrator-crimea.com/ddu0114.html
http://net.pku.edu.cn/~course/cs101/2007/resource/Intro2Algorithm/book6/chap20.htm